# New Tools for visualization of human motion trajectory in quaternion representation


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**Abstract.** The paper presents the results of research on the development of a framework and new tools for perceptually oriented visualization of human motion, in particular, a gait. Presented in this paper are new tools for visualizing motion and gait of a human. Their implementation is based on the following principles: i) translational motion component is omitted and time-varying orientations of individual parts of the body are represented by the trajectories of quaternions, ii) the trajectories of quaternions are visualized using a maps: $S^3 \to \mathbb{R}^3$ implemented as: orthogonal projection, stereographic projection, Hopf transformation. This paper describes only the basic functionalities of the tool, the rest are easy to elicit from the main application screen. Rotations of a rigid body represented by the "3D one" about an axis defined by selected vector are presented as an example of using the tool and to facilitate understanding of motion visualization. Finally the exemplary gait visualisations based on three different maps for healthy subjects and patients with impaired movement are presented. Trial carried out using data from the gait laboratory HML show that the best approach to qualitative and quantitative analysis is based on using orthogonal projection. Other types of maps like Hopf and stereographic are difficult to interpret and may be useful in more specific cases.


## 1 Introduction

In the case of human motion, there are two types of models of human body: i) the skeleton and ii) skeleton-less. The skeleton model is represented by a system of articulated rigid bodies The skeleton-less model is represented by a set of distinguished points and its motion through time. In the rest of this work a skeleton model is used. Each of the rigid bodies of skeleton model is parameterized by the mass and inertia tensor and corresponds to the anatomical body segments like shoulder, arm, foot, hand. Similarly, the number of degrees of freedom for body part connection is equal to the number of degrees of freedom of the respective anatomy joint. In standard models, the number of rigid bodies depends on the assumed model resolution, 22 for Vicon Blade model and 24 for Vicon Nexus model. Specialized models for certain parts of human body do represent those parts with higher resolution, good example of those is clinically tested Oxford Foot Model which uses 3 ridig bodies for foot and 1 for tibia [1]. Configuration of articulated rigid bodies can be described alternatively by: i) definition of position (translation and orientation) of the frame of each body w.r.t. the frame of the world or alternatively w.r.t. the frame of this body in the selected initial configuration (e.g. T pose), ii) definition of the hierarchy tree of bodies and the definition of orientation of the frame body of child w.r.t. current frame. Translation for the frame of child to parent frame is constant due to the assumption of rigidity of bodies. Translation and orientation of the body which is the root of a tree is determined w.r.t. the frame of the world. Regardless of convention of configuration description the key element of visualization is to visualize change in time orientations of individual rigid bodies forming the skeleton. From a formal point of view, the orientation of a rigid body can be parameterized alternatively and equivalently by: i) a $3 \times 3$ skew-symmetric matrix, via exponentiation; the $3 \times 3$ skew-symmetric matrices are the Lie algebra of SO(3), and this is the exponential map readily applicable in Lie theory, ii) Euler angles $(\theta, \varphi, \psi)$, representing a product of rotations about the z, y and z axes, iii) Tait-Bryan angles $(\theta, \varphi, \psi)$, representing a product of rotations about the x, y and z axes, iv) axis angle pair $(\mathbf{n}, \theta)$ of a unit vector representing an axis, and an angle of rotation about it, v) a quaternion q of length 1; the components of which are also called Euler-Rodrigues parameters. The aim of the current work [16,17,18,19,20,21] was to test a wide variety of parameterizations for usability in problems of human motion analysis and classification, including their sensitivity to some of the characteristics features of motion. In this paper quaternion parameterization of orientation was assumed and the goal was to implement and test various techniques of quaternion trajectory visualization and perceptual diagnosis of abnormalities of movement. Tested quaternion visualization techniques uses alternatively: i) orthogonal projection of $S^3$ parallel to the real axis ii) stereographic projection and iii) Hopf map. The developed new tools require experience in the interpretation of the observed trajectory obtained as a result of mapping $S^3 \to \mathbb{R}^3$. For this reason, a separate component of the tool is developed that allows to simulate the trajectory of elementary rotation and to show their visualization.

The literature on visualizing quaternions and more generally quaternions trajectories and fields is extensive, the basic positions are [4] and [5]. Despite of this, diversity and utility of tools implemented on the basis of theoretical concepts of visualization are rather restricted in terms of their funcionalities and perceptual values offered. Most of these like *Meshview* [6], *Quaternion Rotation Demo* [7], *Quaternion - Maps* are authored by A. J. Hanson and serve rather for demonstration than for practical use. Another program of Hanson *Quaternion Demonstrator*, [8], allows the visualization of quaternions maps. J.C. Hart program, [9], [10], allows the visualization of one quaternion simultaneously. There exist also *Quaternion visualization tool in Matlab environment*, [12]. Regardless of the critical evaluation of existing programs visualizing quaternions and quaternions trajectories many of theoretical and graphical ideas included in them were used to create new tools presented in this work.

## 2 Motion Data Acquisition

Gait data used to test the proposed approach and implemented tools have been obtained in the multimodal laboratory-Human Motion Laboratory (HML) of Polish-Japanese Institute of Information Technology
http://www.hml.pjwstk.edu.pl A huge database contains the records of gait motion obtained from healthy patients with coxartroza and movements of people with Parkinson's disease. HML measurements equipment are: 1) Vicon's Motion Kinematics Acquisition and Analysis System equipped with 10 NIR cameras with the acquisition speed of 100 to 2000 frames per second at full frame resolution of 4 megapixels and 8-bit grayscale and acquisition space of 12x7x4 meters. 2) Noraxon's Dynamic Electromyography (EMG) System allowing for 16-channel measurement of muscle potentials with non-gel electrodes in compliance with the SENIAM guidelines. 3) Kistler's Ground Reaction Force (GRF) Measurement System used for measuring ground reaction forces with two dynamometric platforms with measurement ranges adjusted to gait analysis research. The system has a 6-meter path masking two platforms situated in the middle of its length. 4) A system for simultaneous multi-camera video image recording equipped with 4 Basler's cameras that allows for simultaneous image recording from all the cameras in Full HD and lossless video recording. The system uses color video recorders using the GigE Vision standard and industrial lenses. The multimodal motion data available in C3D, AMC/ASF, BVH formats.

## 3 Theoretical Issues

### 3.1 Quaternions

Lets define three distinguished coordinate vectors $(0, 1, 0, 0)$, $(0, 0, 1, 0)$ and $(0, 0, 0, 1)$ named $i$, $j$ and $k$, respectively. The vector $(w, x, y, z)$ written as $q = w + xi + yj + zk$ represents quaternion in algebraic notation. The number $w$ is the real part and $x$, $y$ and $z$ are called the $i$, $j$ and $k$ parts, respectively. Beside algebraic notation we use trygonometric $q = \|q\|(\cos\frac{\theta}{2} + \mathbf{n}\sin\frac{\boldsymbol{\theta}}{\mathbf{2}})$ and exponential $q = \|q\|\exp(\mathbf{n}\theta)$, $\mathbf{n} = \mathbf{0} + \mathbf{n_1 i} + \mathbf{n_2 j} + \mathbf{n_3 k}$ where $n = (n_1, n_2, n_3)$ defines axis of rotation and $\theta$ angle of rotation. Detailed information concerning quaternions can be found in [2,3,11]

The set of unit length quaternions, viewed as points in $\mathbb{R}^4$, is the $3D$-sphere $S^3$. Each nonzero quaternion $q$ has a multiplicative inverse, denoted as $q^{-1}$ and conjugate denoted as $\overline{q}$.

The set $S^3$ with the operation of quaternion multiplication satisfies the axioms of a group. The set of rotations in 3-space, with the operation of composition, is also a group, called $SO(3)$. Each rotation $R$ in $SO(3)$ can be realized by quaternion $q, -q \in S^3$

### 3.2 Orthogonal Projection

Orthogonal projection $S^3 \rightarrow \mathbb{R}^3$ is a basic tool for quaternion trajectory visualisation. Lets represent quaternion as:

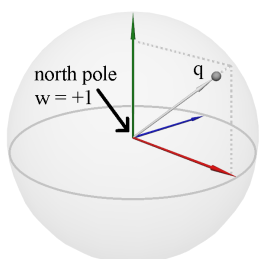
$$\boldsymbol{q} = (w, x, y, z) = (w, \boldsymbol{v}) \tag{1}$$

where:

$$(w)^2 + (x)^2 + (y)^2 + (z)^2 = 1 \tag{2}$$

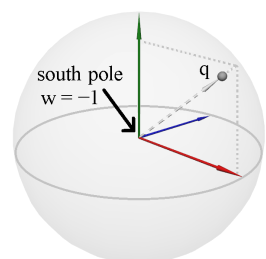
Depending on value of $w$, the set of vectors $v$ can be divided into following subsets; if $w > 0$: north hemisphere, if $w < 0$: south hemisphere and if $w = 0$: equator. Each point of $\mathbf{S^3}$ sphere is projected to a point $(x, y, z)$ in a ball with radius of 1. Value of $w$ can be deduced from $\boldsymbol{v}$ and subset.

$$(w) = \pm\sqrt{1 - \boldsymbol{v} \cdot \boldsymbol{v}} \tag{3}$$




(a) Ball
North hemisphere
$x^2 + y^2 + z^2 < 1$
$0 < w \leq 1$

(b) Sphere
Equator
$x^2 + y^2 + z^2 = 1$
$w = 0$




(c) Ball
South hemisphere
$x^2 + y^2 + z^2 < 1$
$-1 \leq w < 0$

Quaternions have a property of double-covering orientation space. Joints in human body; however, have a limited range of rotations. Therefore, for purpose of visualition, all quaternions from southern hemisphere are reflected onto northern.

$$\boldsymbol{q_{vis}} = \begin{cases} \boldsymbol{q} & w \geq 0 \\ -\boldsymbol{q} & w < 0 \end{cases} \tag{4}$$

Relation between angle of rotation $\theta$ and distance from center of ball $t$ is non-linear and can be described as:

$$t \in < 0; 1 > \tag{5a}$$

$$\theta = 2 \arcsin t \tag{5b}$$

In order to linearize and make visualization more intuitive, following transformation is applied:

$$\boldsymbol{q_{vis}} = (w, x, y, z) = \left(\cos\frac{\theta}{2}, \boldsymbol{n} \sin\frac{\theta}{2}\right) \tag{6a}$$

$$\boldsymbol{P} = \hat{\boldsymbol{n}} \frac{2 \arccos(w)}{\Pi} \tag{6b}$$

where $\boldsymbol{P}$ is transformed position of $\boldsymbol{q_{vis}}$ in visualization space and $\hat{\boldsymbol{n}}$ is normalized $\boldsymbol{n}$ vector.

By applying above formula relation between angle and the distance of P from the centre of $\mathbb{R}^3$ becomes linear. This distance is equal to distance on $S^3$ from $\boldsymbol{q_{vis}}$ to the north pole divided by $\Pi$.

### 3.3 Stereographic Projection

Stereographic projection $S^3/(1,0,0,0) \to \mathbb{R}^3$ considered in this paper as alternative tool for quaternion trajectory visualisation is defined as follows:

$$(w,x,y,z) \to (\frac{x}{1-w}, \frac{y}{1-w}, \frac{z}{1-w}) \tag{7}$$

### 3.4 Hopf Fibration

The Hopf fibration, named after Heinz Hopf, is a useful tool in mathematics and physics and has many physical applications such as quantum information theory [13], magnetic monopoles [14] and rigid body mechanics [15]. In context of this paper a key issue is connection of the Hopf fibration with rotations of $3D$-space. Let $S^3$ and $S^2$ be spheres in $\mathbb{R}^4$ and $\mathbb{R}^3$ respectively The Hopf fibration is the mapping $h$ from $S^3$ to $S^2$ defined as follows:

$$h(w,x,y,z) = (w^2 + x^2 - y^2 - z^2, 2(wz + xy), 2(xz - wy)), \tag{8}$$

under assumption that $q = (w, (x, y, z))$ is quaternion in vector notation.

It can be easily verified that the squares of the three coordinates on the right hand side of (8) sum to 1, so that the image of $h$ is a subset (indeed the whole) of $S^2$.

Hopf fibration map can be also defined in terms of rotation by quaternions. Let $P_0 = (1;0;0)$, be a distinguished point on $S^2$. Let for any given point $(w,x,y,z)$ on $S^3$, $q = w + xi + yj + zk$ be the corresponding unit quaternion. The quaternion $q$ defines a rotation $R_q$ of 3-space. Then the Hopf fibration is defined by:

$$q \to R_q(P_0) = qi\bar{q} \tag{9}$$

Consider once again the point $P_0 = (1,0,0)$ in $S^2$. One can easily check that the set of points $C = (cost, sint, 0, 0), t \in \mathbb{R}$ or in other representation; a rotation about axis defined by vector $(1,0,0)$, map to (1; 0; 0) via the Hopf fibration $h$. In this case set $C$ is the entire set of points that map to $(1,0,0)$ via h. In other words, $C$ is the preimage set $h^{-1}(1,0,0)$. The set $C$ is the unit circle in a plane in $\mathbb{R}^4$. In general for any point $P$ in $S^2$, the preimage set $h^{-1}(P)$ is a circle in $S^3$ which is called the fiber of the Hopf map for P. Let us note that the converse statement is not true. For example the circles in $S^3$ defined as $C = (cost, 0, sint, 0)$, $C = (cost, 0, 0, sint)$ and representing rotations about axis defined by vectors $(0,1,0)$,$(0,0,1)$ are mapped to circles in $\mathbb{R}^2$.

# 4 General Tool Description

Tool developed for quaternion visualization was implemented using .NET framework and is based on Windows Forms technology. There are two distinct applications of which each presents different frontend, however both utilize same implementation of all transformations to achieve its goals of visualizing quaternions that can be selected using GUI. Visualizations are shown in separate panels, each providing controls for specific visualization. All visualizations can be saved to PNG file at any given time using current orientations of coordinate systems selected in each visualization panel.

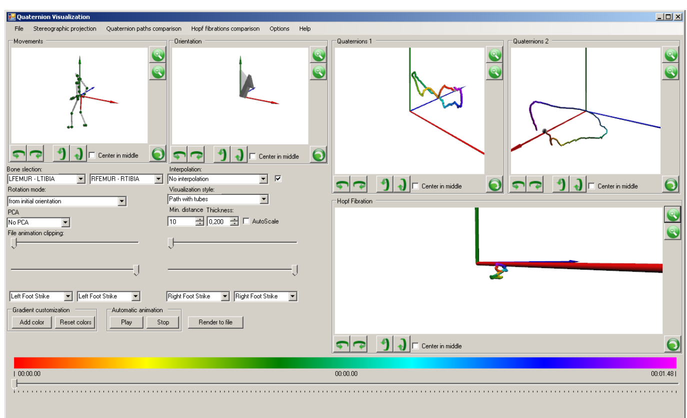
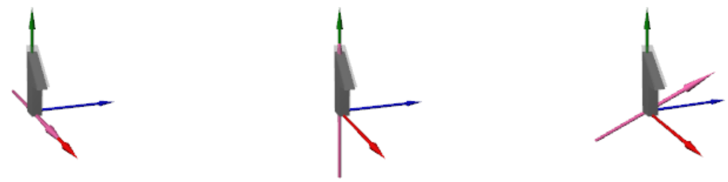
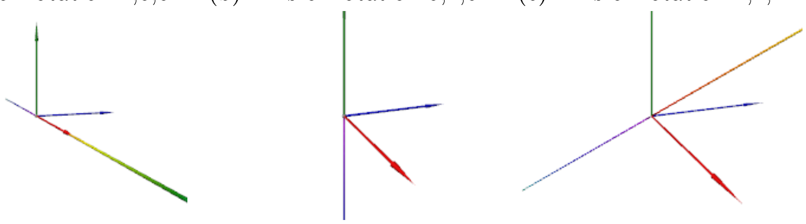



Fig. 1: Main screen of QuaternionVisualization application

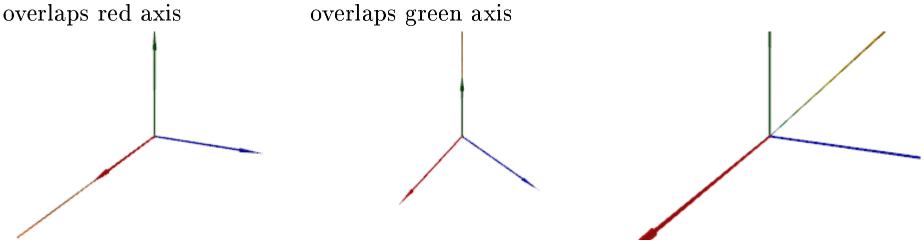
First application, QuaternionVisualization has ability to load skeleton-based animation files containing information about rotation of each segment of human body around its parent joint defined by skeleton. Additional data, like time markers related to specific events (e.g. beginning and end of each step) can also be loaded from C3D files. In each frame there is a visualization of quaternion in time using techniques mentioned. Data from animation file can be clipped using arbitrary values for point in time in which to begin and end visualization, or it can be selected from events provided by loaded C3D files. Two different segments can be selected at once in order to compare them on separate or combined screen.



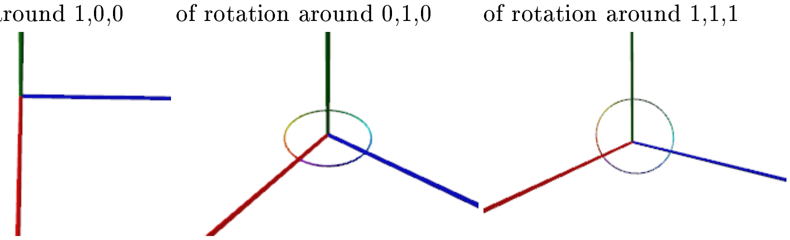
(a) Axis of rotation 1,0,0 (b) Axis of rotation 0,1,0 (c) Axis of rotation 1,1,1



(d) Orthogonal projection of rotation around 1,0,0, which overlaps red axis (e) Orthogonal projection of rotation around 0,1,0, which overlaps green axis (f) Orthogonal projection of rotation around 1,1,1



(g) Stereographic projection of rotation around 1,0,0 (h) Stereographic projection of rotation around 0,1,0 (i) Stereographic projection of rotation around 1,1,1



(j) Hopf fibration of rotation around 1,0,0 (k) Hopf fibration of rotation around 0,1,0 (l) Hopf fibration of rotation around 1,1,1

Fig. 2: Sample visualizations using QuaternionVisualizationLearning application

Second application, QuaternionVisualizationLearning, exists to prove correctness of implementation and to show how each type of visualization works for typical data. User can select axis of rotation and an angle and create a list of such rotations that will be visualized together as one path. User can also customize visualization to suit his needs by selecting visualization style, visibility of objects, thickness of lines, etc. User can select specific quaternion using slider to see which part of visualization is based on selected quaternion. A shape of **1** is used to show orientation and rotation axis of provided data.

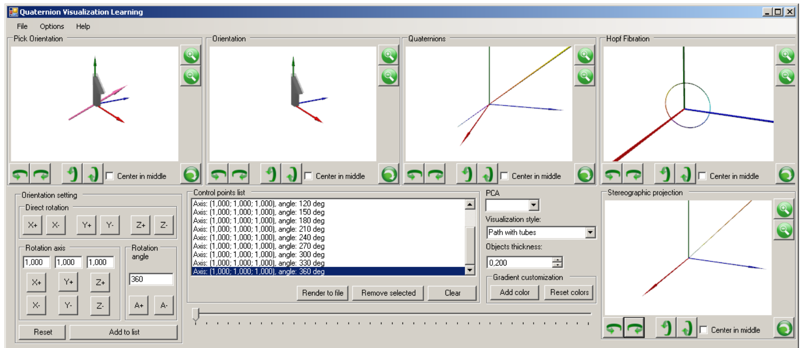
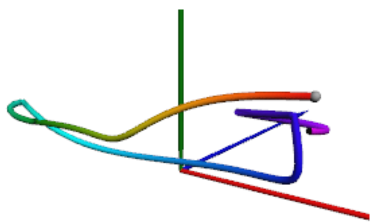
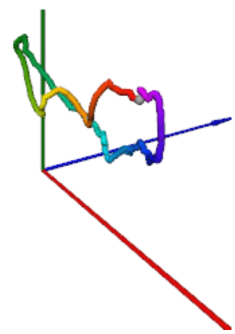



Fig. 3: Main screen of QuaternionVisualizationLearning application

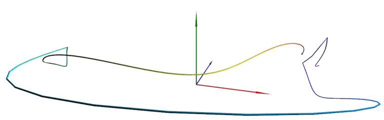
## 5 Exemplary results



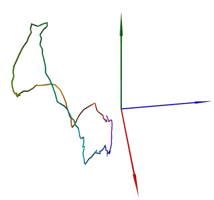
(a) Healthy patient - orthogonal projection



(b) Sick patient - orthogonal projection



(c) Healthy patient - stereographic projection



(d) Sick patient - stereographic projection

(e) Healthy patient - Hopf fibration

(f) Sick patient - Hopf fibration

Fig. 4: Exemplary results collected from healthy and sick patients

## 6 Conclusions and Further Works

Visualizations underlying the following comparison and summary refer to the case of hierarchical representation of pose as a ordered set of child body - parent body rotations. Other cases discussed in the first chapter are also acceptable and supported by the software, but their interpretation is more difficult because of the possible change of orientation of the entire skeleton. Visualizations are made based on the following three different techniques of mapping $S^3$ sphere onto $\mathbb{R}^3$: i) the orthogonal projection which maps the northern hypersphere of $S^3$ sphere on the $B^3$ ball, ii) the stereographic projection which maps the northern hypersphere of $S^3$ sphere to $\mathbb{R}^3$, and finally iii) Hopf map $S^3$ to $S^2$. Orthogonal projection is easiest to perceive because in this case the quaternion encoding rotation is visualized by the vector defined rotation axis. The length of this vector is scaled linearly or non-linearly by the size of the angle of rotation. Such visualization is perceptually clear in the case of the regular gait in which the axes of rotation of each rigid body is approximately constant. The variability of the axes may be an important prerequisite for diagnostic reason. For example, in the case of the knee, due to the anatomical structure of this joint axis of rotation performs precession. The shape and size of the surface generated by the vector of rotation axis in some cases approximated by a cone of precession are important for diagnosis. The disadvantage of visualization techniques based on the orthogonal projection is the difficulty in visualizing the time. In the developed software time is visualized by color of the trajectory point, plus a separate time line. The difficulty of visualizing time causes difficulties in reading, even qualitative, angular velocity. Although there is a possibility of visualizing the points of trajectory by markers or numbers, experiments performed have shown that this leads to decreasing perceptual quality of visualization. As seen in fig. 4, one could assess health and sick patient's movement based on shakiness of trajectory. Interpretation of the stereographic projection is based on knowledge of the images of selected arcs on the sphere $S^3$. Of particular interest is the visualization based on the Hopf map. As shown in section 2 rotation with respect to the axis defined by the vector (1,0,0) maps to a point (1,0,0). Consequently, the "scatter" around the point (1,0,0) on the sphere $S^2$ may be a measure how much the actual rotation about actual axis is different from the ideal rotation with respect to axis defined by vector (1,0,0). Still though, analysis of the utility of visualization based on Hopf map technology requires further research. Therefore, one objective of the work is wide dissemination of software in order to obtain opinions of its usefulness.

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