Phase Space Reconstruction and Estimation of the Largest Lyapunov Exponent for Gait Kinematic Data

Henryk Josiński[*], Adam Świtoński[†,*], Agnieszka Michalczuk[†] and Konrad Wojciechowski[†]

[*]*Silesian University of Technology, Akademicka 16, 44-100 Gliwice, Poland*
[†]*Polish-Japanese Institute of Information Technology, Aleja Legionów 2, 41-902 Bytom, Poland*

Abstract. The authors describe an example of application of nonlinear time series analysis directed at identifying the presence of deterministic chaos in human motion data by means of the largest Lyapunov exponent. The method was previously verified on the basis of a time series constructed from the numerical solutions of both the Lorenz and the Rössler nonlinear dynamical systems.

Keywords: phase space reconstruction, deterministic chaos, human motion analysis
PACS: 05.45.Pq, 05.45.Tp

INTRODUCTION

Dynamics properties of a system can be determined on the basis of its model (provided that it is known) consisting of differential or difference equations or through analysis of experimental data collected as result of system observation.

The state of a dynamical system at a given instant of time can be represented by a point in the phase space spanned by the state variables of the system. Many nonlinear or infinite-dimensional dynamical systems exhibit chaotic behavior. The presence of deterministic chaos is characterized by extreme sensitivity to initial conditions. This hallmark means that initially nearby points can evolve quickly into very different states. In case of analysis of experimental data, fundamental components of the process of determining existence of chaos in a signal represented by a time series are phase space reconstruction and subsequent estimation of the Lyapunov exponents which quantify the average exponential rate of divergence of initially nearby phase space trajectories [1]. Thus, a positive value of the largest Lyapunov exponent (LLE) implies chaotic behavior.

Chaoticity was observed in a variety of systems from several areas including, among others, meteorology, physics, engineering, economics and biology. From among biomedical signals EEG, ECG and gait kinematic data are worthy to note.

The chaotic characteristics of the ECG signals (Lyapunov exponents spectrum and correlation dimension) were incorporated to the set of features for the purpose of biometric individual identification [2] but, first of all, chaos theory has been applied to the analysis of electrocardiogram for examination of cardiac disorders [3].

Chaos is also present in epileptic EEG signals. Brain activity during seizure differs greatly from that of normal state which can be observed as a decrease in chaoticity in the minutes before the seizure. Thus, analysis of the changes of the LLE allows for the detection and prediction of the incoming epileptic seizure [4], [5].

In [6], [7] LLE was estimated to quantify the local dynamic stability (LDS) of human walking kinematics, that is to say, the degree of resilience of gait control to small perturbations [8].

The research described in the present abstract aimed at verification of the complex procedure of computing the LLE on the basis of a time series constructed from the numerical solution for well-known nonlinear dynamical systems (Lorenz, Rössler [9]) with a view to its subsequent application for the purpose of identification of deterministic chaos in gait kinematic data for patients suffering from various diseases affecting way of walking.

METHOD OF THE TIME SERIES ANALYSIS

Nonlinear time series analysis methods enable the determination of characteristic invariants such as the LLE of a particular system solely by analyzing the time course of one of its variables [10]. Nevertheless, identification of chaotic behavior based on experimental data is a multistage process. The first step constitutes a phase space reconstruction. On the basis of Takens' embedding theorem [11] the phase space can be reconstructed using time-delayed measurements of a single observed signal in form of a time series. Reconstruction consists in viewing a time series $x_k = x(k\tau_s), k = 1, \ldots, N$ in a Euclidean space $\mathbb{R}^m$, where m is the embedding dimension and τ_s is the sampling time [12]. Each m-dimensional embedding vector is formed as $\mathbf{x}_k = [x_k, x_{k+\tau}, x_{k+2\tau}, \ldots, x_{k+(m-1)\tau}]^T$, where τ is the delay time. The selection of τ and m is important for the sake of reconstruction quality. It is worthwhile to mention that the properties associated with the system's dynamics (inter alia Lyapunov exponents) are preserved in the new phase space.

Time delay τ was calculated from the first local minimum of the mutual information function (MI). Mutual information between x_t and $x_{t+\tau}$ is a measure of how much information can be predicted about one time series point given full information about the other [1]. Assuming that the range of values in a time series was partitioned into j intervals of equal length, the mutual information function $I(\tau)$ can be computed according to the following formula:

$$I(\tau) = \sum_{h=1}^{j} \sum_{k=1}^{j} P_{h,k}(\tau) \log_2 \left(\frac{P_{h,k}(\tau)}{P_h P_k} \right) \tag{1}$$

where h and k are indices of intervals, P_h, P_k denote the probabilities that x_t assumes a value within the h-th, k-th interval, respectively, and $P_{h,k}(\tau)$ is the joint probability that x_t belongs to the h-th interval and $x_{t+\tau}$ is taken from the k-th interval.

The minimal m that is required to fully resolve the structure of the system in the reconstructed phase space was found by the method of "False Nearest Neighbors" (FNN) [13], which is based on the following assumption constituting the condition of no self-intersections – if the attractor (i.e. a set of states towards which neighboring states asymptotically approach in the course of dynamic evolution [14]) is to be reconstructed successfully in $\mathbb{R}^m$, then all points that are close in $\mathbb{R}^m$ should also be sufficiently close in $\mathbb{R}^{m+1}$ [12]. A point which does not satisfy this condition is a "false" neighbor. The number of such points is computed for increasing embedding dimension until the percentage of "false" neighbors is below a given threshold.

The mean divergence between neighboring trajectories in the phase space at time t is described by the following formula:

$$d(t) = De^{\lambda_1 t} \tag{2}$$

where D is the initial separation between neighboring points and λ_1 is the LLE [6]. The Rosenstein algorithm [15] estimates the LLE locating nearest neighbors on adjacent trajectories and computing the divergence between successive pairs along the trajectories. On the basis of the formula (2) the Euclidean distance $d_j(i)$ between the j-th pair of nearest neighbors after i time steps of the length equal to Δt and the LLE are linked in the following way:

$$\ln [d_j(i)] \approx \lambda_1 (i \cdot \Delta t) + \ln [D_j] \tag{3}$$

Hence, the LLE is estimated as the slope of the average logarithmic divergence of the neighboring trajectories:

$$y(i) = \frac{1}{\Delta t} \langle \ln [d_j(i)] \rangle \tag{4}$$

where $\langle \cdot \rangle$ denotes the average over all values of j.

NUMERICAL EXPERIMENTS

In the first stage of the research, for both the Lorenz and the Rössler systems a time series was constructed from the numerical solution for the single state variable sampled from $t_0 = 0$ to $t_{max} = 100$ at intervals $\Delta t = 0.01$. Parameters of the model (i.e. coefficients of the first-order system of state equations), initial conditions of the state variables, parameters of the phase space reconstruction and computed LLE values (the "LLE (Rosenstein)" column) were collected in Table 1. The results are consistent with LLE values obtained by means of the Wolf algorithm [16] using the Jacobian matrix constructed on the basis of state equations (the "LLE (Wolf)" column).

The next stage of the research consisted in analysis of gait sequences which were recorded in the Human Motion Laboratory (HML) of the Polish-Japanese Institute of Information Technology [17] by means of the Vicon Motion Kinematics Acquisition and Analysis System. The Vicon system is equipped with 10 NIR (Near InfraRed) cameras

TABLE 1. Description of the experiments with dynamical systems

System	Model parameters	Initial conditions	Time delay τ	Embedding dimension m	LLE (Rosenstein)	LLE (Wolf)
Lorenz	$\sigma = 10\ \ \rho = 28\ \ \beta = 8/3$	$x_0 = 1\ \ y_0 = z_0 = 0$	17	3	0.7654	0.7164
Lorenz	$\sigma = 16\ \ \rho = 45.92\ \ \beta = 4$	$x_0 = 1\ \ y_0 = z_0 = 0$	11	3	1.4670	1.4118
Rössler	$a = b = 0.2\ \ c = 5.7$	$x_0 = 1\ \ y_0 = z_0 = 0$	128	3	0.0757	0.0749

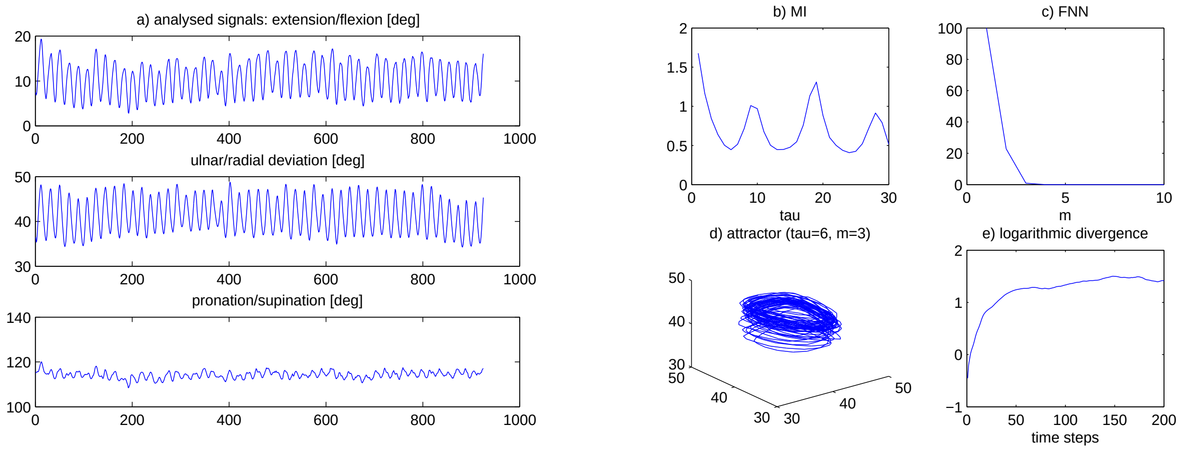

FIGURE 1. Stages of the LLE computation: a) analysed signals, b) mutual information, c) percentage of false nearest neighbors, d) reconstructed attractor, e) logarithmic divergence

recording the movement of an actor wearing a special suit with attached markers (the *motion capture* process). Positions of the markers in consecutive time instants constitute basis for reconstruction of their 3D coordinates. The gait route was specified as a 5 meters long straight line (use of a treadmill is planned for future recordings).

The experiment described in the present abstract was aimed at investigating chaotic behavior in tremor-affected movements of both wrists of a patient suffering from Parkinson's disease (PD). A single analysed time series represented one of the following types of wrist movement at the joint: up and down – *extension/flexion* (Ext/Fl), sidewards – *ulnar/radial deviation* (Ul/Rad), and rotation – *pronation/supination* (Pr/Sup) [18]. Fig. 1 illustrates results of successive stages of the LLE computation for the right wrist: a) analysed signals, b) mutual information, c) percentage of false nearest neighbors, d) reconstructed attractor, e) divergence. Subfigures b)-e) were created for the case of ulnar/radial deviation.

Table 2 includes the LLE values for all types of wrist movement for the patient suffering from PD as well as for a healthy person.

In contrast to the results for the healthy person, all types of wrist movement for the patient suffering from PD are characterized by the positive LLE values which indicate the presence of deterministic chaos. The noticeable difference in the LLE values between both wrists of the PD patient confirmed conclusions drawn from visual observation of him. All computations were performed using MATLAB.

TABLE 2. The largest Lyapunov exponents for both examined persons

Type of movement	Left wrist (PD)	Right wrist (PD)	Left wrist (Healthy)	Right wrist (Healthy)
Ext/Fl	0.1233	1.0460	-1.0136	-0.1383
Ul/Rad	0.0750	2.2593	-0.0782	-0.1645
Pr/Sup	0.1544	0.4440	-0.0856	-0.0743

CONCLUSION

The authors described an example of identification of the presence of chaotic behavior in human motion data based on phase space reconstruction and estimation of the LLE. The applied procedure of a time series analysis will be extended by incorporation of other measures, such as correlation dimension and approximate entropy, in the hope that it will constitute support for assessment of gait disorders (among others resulting from PD, stroke, osteoarthritis of the hip or osteoarthritis of the spine).

ACKNOWLEDGMENTS

This work was supported by the project DEC-2011/01/B/ST6/06988 from the Polish National Science Centre.

REFERENCES

1. B. Henry, N. Lovell and F. Camacho, “Nonlinear Dynamics Time Series Analysis” in *Nonlinear Biomedical Signal Processing: Dynamic Analysis and Modeling*, edited by M. Akay, Wiley Online Library, published online 2012, pp. 1–39.
2. Ch.-K. Chen, Ch.-L. Lin and Y.-M. Chiu, “Individual Identification Based on Chaotic Electrocardiogram Signals”, 6th IEEE Conference on Industrial Electronics and Applications, 2011, pp. 1765–1770.
3. M. E. Cohen, *Chaos*, Wiley Encyclopedia of Biomedical Engineering, 2006.
4. S. Osowski, B. Świderski, A. Cichocki and A. Rysz, “Epileptic seizure characterization by Lyapunov exponent of EEG signal”, *COMPEL: The International Journal for Computation and Mathematics in Electrical and Electronic Engineering* **26**(5), 1276–1287 (2007).
5. F. Mormann, R. G. Andrzejak, Ch. E. Elger and K. Lehnertz, “Seizure prediction: the long and winding road”, *Brain*, **130**, 314–333 (2007).
6. J. B. Dingwell and J. P. Cusumano, “Nonlinear time series analysis of normal and pathological human walking”, *Chaos* **10**(4), 848–863 (2000).
7. J. B. Dingwell and L. C. Marin, “Kinematic variability and local dynamic stability of upper body motions when walking at different speeds”, *Journal of Biomechanics* **39**, 444–452 (2006).
8. Ph. Terrier and O. Deriaz, “Non-linear dynamics of human locomotion: effects of rhythmic auditory cueing on local dynamic stability”, *Frontiers in Physiology* **4**, 1–13 (2013).
9. J. Awrejcewicz and R. Mosdorf, *Numerical Analysis of Some Problems of Chaotic Dynamics*, WNT, Warsaw, 2003.
10. M. Perc, “The dynamics of human gait”, *European Journal of Physics* **26**, 525–534 (2005).
11. F. Takens, “Detecting Strange Attractor in Turbulence”, *Lecture Nodes in Mathematics* **898**, 366–381 (1981).
12. D. Kugiumtzis, “State Space Reconstruction Parameters in the Analysis of Chaotic Time Series – the Role of the Time Window Length”, *Physica D: Nonlinear Phenomena* **95**(1), 13–28 (1996).
13. M. B. Kennel, R. Brown and H. D. I. Abarbanel, “Determining embedding dimension for phase-space reconstruction using a geometrical construction”, *Physical Review A* **45**(6), 3403–3411 (1992).
14. E. W. Weisstein, “Attractor” From MathWorld – A Wolfram Web Resource: `http://mathworld.wolfram.com/Attractor.html`.
15. M. T. Rosenstein, J. J. Collins and C. J. De Luca, “A practical method for calculating largest Lyapunov exponents from small data sets”, *Physica D* **65**, 117–134 (1993).
16. A. Wolf, J. B. Swift, H. L. Swinney and J. A. Vastano, “Determining Lyapunov exponents from a time series”, *Physica D* **16**, 285–317 (1985).
17. Webpage of the Human Motion Laboratory of the Polish-Japanese Institute of Information Technology: `http://hm.pjwstk.edu.pl`.
18. P. W. Johnson, P. Jonsson and M. Hagberg, “Comparison of measurement accuracy between two wrist goniometer systems during pronation and supination”, *Journal of Electromyography and Kinesiology* **12**, 413–420 (2002).