Recurrence Plots and Recurrence Quantification Analysis of Human Motion Data

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Abstract. The authors present exemplary application of recurrence plots, cross recurrence plots and recurrence quantification analysis for the purpose of exploration of experimental time series describing selected aspects of human motion. Time series were extracted from treadmill gait sequences which were recorded in the Human Motion Laboratory (HML) of the Polish-Japanese Academy of Information Technology in Bytom, Poland by means of the Vicon system. Analysis was focused on the time series representing movements of hip, knee, ankle and wrist joints in the sagittal plane.

Keywords: recurrence plot, recurrence quantification analysis, time series, human motion analysis.
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INTRODUCTION

Recurrence plots (RP) introduced by Eckmann *et al.* [1] and recurrence quantification analysis (RQA) developed by Zbilut and Webber [2] have become important tools for analysis of dynamical systems. The goal of the present paper is to present their application to exploration of experimental time series describing selected aspects of human motion.

A dynamical system is any system that evolves in time [3]. The state of a dynamical system at a given instant of time can be represented by a point in the phase space spanned by the state variables of the system. The term "recurrence of states" refers to the situation in which states in a phase space become arbitrarily close to each other after some time. As a fundamental property of dissipative dynamical systems, recurrence of states is typical for nonlinear or chaotic systems. The system will return to close proximity of the former state even though in the meantime small perturbations cause exponential divergence of the states [4].

Characteristic invariants of a particular system can be determined solely by analyzing the time course of one of its variables. On the basis of Takens' embedding theorem [5] the phase space can be reconstructed using time-delayed measurements of a single observed signal in form of a time series. Reconstruction consists in viewing a time series $x_k = x(k\tau_s), k = 1, \dots, Z$ in a Euclidean space $\mathbb{R}^m$, where m is the embedding dimension and τ_s is the sampling time [6]. Each m-dimensional embedding vector is formed as $\mathbf{x}_l = \left[x_l, x_{l+\tau}, x_{l+2\tau}, \dots, x_{l+(m-1)\tau}\right]^T$, where τ is the delay time. Elements of the embedding vector $\mathbf{x}_l$ determine coordinates of the point X_l on the trajectory in the reconstructed phase space. Length of the trajectory is equal to $N = Z - (m-1)\tau$.

RECURRENCE PLOTS AND RECURRENCE QUANTIFICATION ANALYSIS

Recurrence plots constitute a method of visualization of the recurrence of states in a phase space by means of symmetrical $N \times N$ matrices in which element R_{ij} is defined as follows:

$$R_{i,j} = \begin{cases} 1, & \left\| X_i - X_j \right\| \le \varepsilon \\ 0, & \left\| X_i - X_j \right\| > \varepsilon \end{cases} \quad 1)$$

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In other words, a value of 1 is assigned to the element R_{ij} whenever a point X_i on the trajectory is close enough to another point X_j [7]. Thus, a trajectory from *m*-dimensional phase space can be analyzed by means of 2-dimensional representation of its recurrences. The parameter $\varepsilon, \varepsilon \geq 0$ is called the *recurrence threshold*. Exemplary recurrence plot for the time series representing the periodic function $y(t) = \cos(t/2) + \sin(t) + \sin(2t)$ along with the function graph is shown in Fig. 1. The sampling time was equal to 0.01, the phase space reconstruction parameters were as follows: $m = 3, \tau = 32$ and the recurrence threshold $\varepsilon = 0.1$.

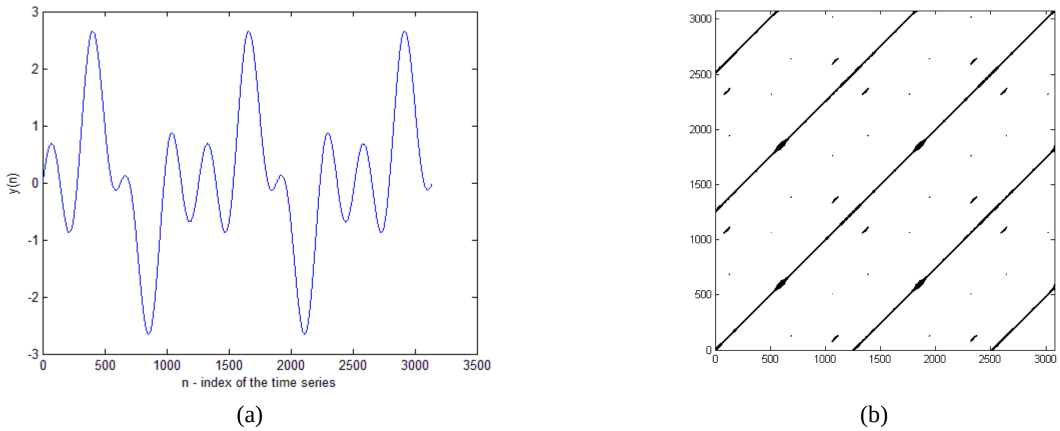


FIGURE 1. (a) Graph of a periodic function, (b) the corresponding recurrence plot.

Diagonal lines in Fig. 1b denote that the evolution of states is similar at different time instants. Furthermore, equal distance between lines confirms the periodic character of the time series.

Authors of the RQA proposed a set of measures quantifying features of the RPs. *Recurrence rate* (RR) is a density of recurrence points in an RP:

$$RR = \frac{1}{N^2} \sum_{i,j=1}^{N} R_{i,j} \quad 2)$$

Determinism (DET) is a quota of recurrence points that form diagonal lines:

$$DET = \frac{\sum_{l=l_{\min}}^{N} l \cdot P(l)}{\sum_{i,j=1}^{N} R_{i,j}} \quad 3)$$

where $P(l)$ is the histogram of the lengths l of diagonal lines and l_{min} denotes the minimal length of such lines.

Another measures determine, *inter alia*, a quota of recurrence points that form vertical lines (*laminarity*; LAM), averaged length of diagonal lines (⟨L⟩), average length of vertical lines (*trapping time*; TT), length of the longest diagonal line (L_{max}) excluding the main diagonal which is called *line of identity* (LoI), length of the longest vertical line (V_{max}), reciprocal of the L_{max} (*divergence*; DIV), Shannon entropy of the distribution of the diagonal lines lengths (ENTR).

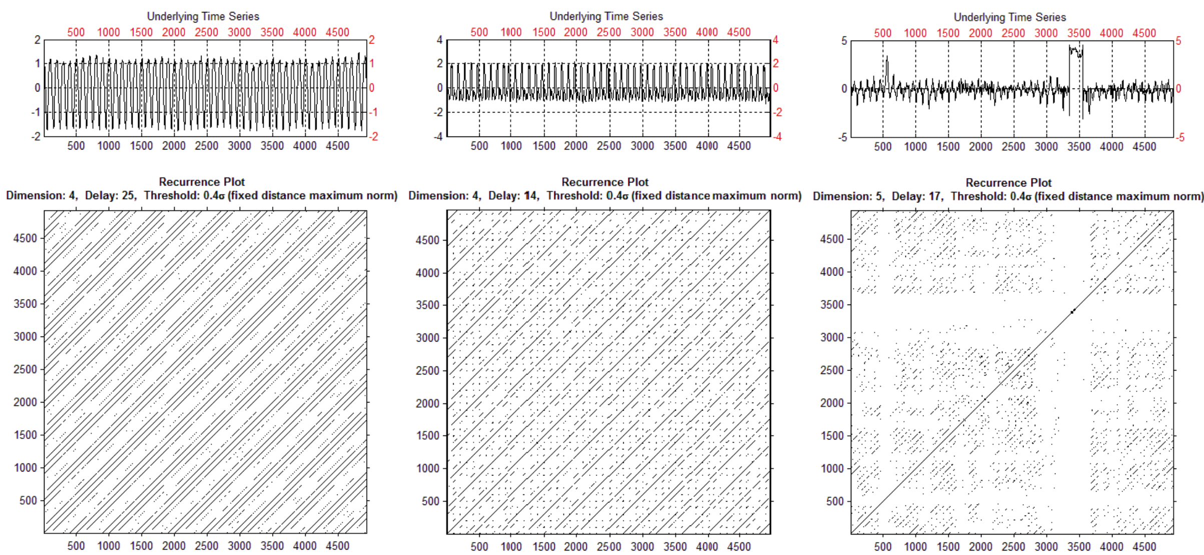
Idea of RP was expanded by Zbilut and Webber to the concept of cross recurrence plots (CRP), which enable to compare the dynamic behavior of two time series embedded in phase space [7].

EXPERIMENTAL RESULTS

For the purpose of time series analysis, gait kinematic data should be acquired in a continuous way over many cycles of motion. A treadmill is very useful in this regard. The analyzed gait sequences of continuous walking of length of several dozen seconds were recorded with a frequency of 100 Hz in the Human Motion Laboratory (HML) of the Polish-Japanese Academy of Information Technology (*http://hm.pjwstk.edu.pl*). The Vicon Motion Kinematics Acquisition and Analysis System which was used for recordings is equipped with 10 NIR (Near InfraRed) cameras registering movements of a subject wearing a suit with attached markers (the *motion capture* process). Positions of the markers in consecutive time instants constitute basis for reconstruction of their 3D coordinates.

Recorded time series represent angles of ankle, knee, hip and wrist joints' movements in sagittal plane which divides body into left/right parts. Next, parameters of a phase space reconstruction were determined independently for each time series – time delay τ was calculated from the first local minimum of the mutual information function (MI) [3] and embedding dimension *m* was found by the method of "False Nearest Neighbors" (FNN) [8]. Value of recurrence threshold ε was selected taking into account one from the "rules of thumb" collated in [9], according to which ε should not exceed 10% of the mean phase space diameter. All computations were performed in the MATLAB environment. Recurrence plots along with the RQA were prepared using the CRP Toolbox [10].

Figures 2a-c include recurrence plots based on the time series representing movements of left hip, left knee and left wrist joints, respectively, recorded for the same subject as part of the same gait sequence.



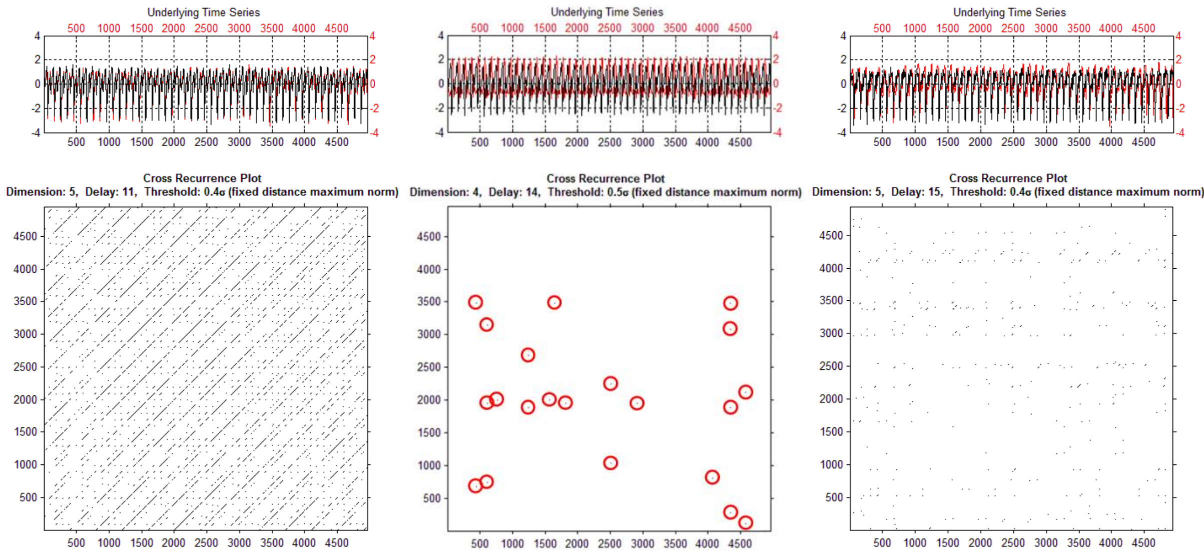
(a) (b) (c)

FIGURE 2. Recurrence plots: (a) left hip joint, (b) left knee joint, (c) left wrist of the same subject.

Values of aforementioned RQA measures are as follows: a) RR = 0.075, DET = 0.997 (hip joint), b) RR = 0.057, DET = 0.981 (knee joint), c) RR = 0.024, DET = 0.892 (wrist). Two first cases satisfy to some degree the following rule: "if DET ≈ 1for a very small recurrence rate (RR < 0.05), then the underlying system will be a deterministic one" [10], which is confirmed by domination of diagonal line segments. By contrast, white bands in Fig. 2c indicate states that are rare and untypical (unintentional hand movement). The lack of vertical and horizontal lines reveals absence of states that do not change or change slowly for some time.

Cross recurrence plots in Fig. 3a-c represent 3 cases: a) 2 gait sequences of the same subject – left ankle joint vs left ankle joint, b) 2 joints of the same subject – left ankle joint vs left knee joint, c) 2 subjects – right ankle joint vs right ankle joint. In each case values of the parameters of phase space reconstruction (*m*, τ) were the same for both compared time series.

Relatively long diagonal structures in Fig. 3a show similar phase space behavior of both time series (as expected in case of movements of the same joint of the same subject but stemming from different recordings). In contrast, single isolated points in Fig. 3b (surrounded by red circles for the sake of better visibility) indicate almost complete dissimilarity of the time series representing movements of different joints of the same subject (also as expected). Finally, small clusters of several points in Fig. 3c show that in case of the same type of movements of the same joint but performed by two subjects both compared time series are characterized by greater mutual similarity than in previously discussed case (Fig. 3b), even if the recurrence threshold ε for Fig. 3b is greater than its counterpart for Fig. 3c.



(a) (b) (c)

FIGURE 3. Cross recurrence plots: (a) 1 subject, left ankle joint, different gait sequences, (b) 1 subject, left ankle joint vs left knee joint, (c) 2 subjects, right ankle joint.

CONCLUSION

Due to space limitation the present abstract includes only exemplary recurrence plots for human motion data, and the recurrence quantification analysis of their features is merely fragmentary. Nevertheless, it might be said that both tools seem to be very useful for the purpose of motion data analysis, especially together with other methods intended for examination of dynamical invariants of a system (e.g. the largest Lyapunov exponent).

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REFERENCES

1. J.-P. Eckmann, S. O. Kamphorst, D. Ruelle, “Recurrence plot of dynamical systems”, *Europhysics Letters*, **4**(9), 1987, pp. 973-977.
2. Ch. L. Webber Jr., J. P. Zbilut, “Dynamical assessment of physiological systems and states using recurrence plot strategies”, *Journal of Applied Physiology*, **76**(2), 1994, pp. 965-973.
3. B. Henry, N. Lovell, F. Camacho, “Nonlinear Dynamics Time Series Analysis”, in *Nonlinear Biomedical Signal Processing: Dynamic Analysis and Modeling*, Vol. 2, Wiley Online Library, published online 2012, pp. 1-39.
4. N. Marwan, J. Kurths, “Cross Recurrence Plots and Their Applications”, *Mathematical Physics Research at the Cutting Edge*, Nova Science Publishers, Inc., 2004, pp. 101-139.
5. F. Takens, “Detecting Strange Attractor in Turbulence”, *Lecture Nodes in Mathematics*, **898**, 1981, pp. 366-381.
6. D. Kugiumtzis, “State Space Reconstruction Parameters in the Analysis of Chaotic Time Series – the Role of the Time Window Length”, *Physica D: Nonlinear Phenomena*, **95**(1), 1996, pp. 13-28.
7. J. P. Zbilut, Ch. L. Webber Jr., “Recurrence quantification analysis”, *Wiley Encyclopedia of BiomedicalEngineering*, John Wiley & Sons, Inc., 2006, DOI: 10.1002/9780471740360.edb1355.
8. M. B. Kennel, R. Brown, H. D. I. Abarbanel, “Determining embedding dimension for phase-space reconstruction using a geometrical construction”, *Physical Review A*, **45**(6), 1992, pp. 3403-3411.
9. N. Marwan, “How to avoid potential pitfalls in recurrence plot based data analysis”, *International Journal of Bifurcation and Chaos*, **21**(4), World Scientific Publishing Company, 2011, pp. 1003-1017.
10. N. Marwan, M. C. Romano, M. Thiel, J. Kurths, “Recurrence Plots for the Analysis of Complex Systems”, *Physics Reports*, **438**(5-6), 2007, pp. 237-329.